Homework I.

① Show that $SU(2)$ (2×2 unitary matrices with determinant $=1$) is both connected and compact.

Hint: Show that $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Apply that in $SU(2)$ case, keeping in mind that the unitarity condition $U^{+}U = 1$ is equivalent to $U = (U^{+})^{-1}$

Use that to find homeomorphism between $SU(2)$ and a topological space you know very well.

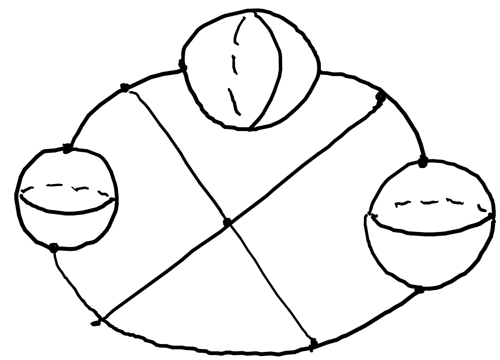
② Show that homotopy is compatible with composition:

If $f, g: X \to Y$ are homotopic, and $f', g': Y \to Z$ are homotopic, then so are $f'f: X \to Z$, $g'g: X \to Z$

(3) Show that a retract of a contractible space is contractible.

(4) Use van Kampen theorem to find the fundamental group of the space, obtained by glueing two Möbius bands along their boundary

(5) Calculate the fundamental group of the following space:



(6) Classify all CW-complexes with two 0-cells and two 1-cells up to a) homeomorphism

b) homotopy equivalence

7. Let X be the quotient space of S^2, obtained by identifying north and south poles.
Put a cell complex structure on X and use it to compute $\pi_1(X)$.

8. Let $X \subset \mathbb{R}^3$ be the union of n lines through the origin. Compute $\pi_1(\mathbb{R}^3 \setminus X)$