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Short communication
Quantization of integrable models with hidden symmetries: super-KdV equation

PETR P. KULISH† and ANTON M. ZEITLIN‡

†St. Petersburg Department of Steklov Mathematical Institute, Fontanka, 27, St. Petersburg, 191023, Russia
‡Department of High Energy Physics, Physics Faculty, St. Petersburg State University, Ul'yanovskaja 1, Petrodvoretz, St. Petersburg, 198904, Russia; email: zam@math.ipme.ru, http://www.ipme.ru/zam.html

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Nonlinear integrable systems are very important in superstring theory, quantum and condensed matter physics. It is very hard to quantize such systems due to their nonlinearity and complicated algebraic and geometric structure. We consider some features of their quantization in the case of the supersymmetric Korteweg–de Vries equation (sKdV) [1, 2]. The sKdV hierarchy is based on the $\widehat{osp}(1|2)$ affine superalgebra, which gives us the possibility to include fermion fields in the model. Being an infinite-dimensional integrable system, the sKdV equation possesses an infinite number of integrals of motion (IM) forming an involutive set according to the Poisson bracket. The Poisson structure is given by the functional realization of the superconformal algebra. The so-called Miura transformation gives us the possibility to work with the model in terms of free fields: bosonic and fermionic ones. Our system allows the Lax ($\mathcal{L}-\mathcal{A}$) pair formulation. Considering the auxiliary linear problem for the $\mathcal{L}$ operator, one can write the expression for the monodromy matrix for its solution in terms of a P-exponent. The logarithm of the supertrace of the monodromy matrix will generate the involutive set of local IM. Making a shift in the expression for the monodromy matrix one obtains the auxiliary $\mathbf{L}^{(\mathrm{cl})}$ matrices, which satisfy the following quadratic Poisson bracket relation:

$$\left\{\mathbf{L}_s^{(\mathrm{cl})}(\lambda) \otimes, \mathbf{L}_{s'}^{(\mathrm{cl})}(\mu)\right\} = \left[\mathbf{r}_{ss'}(\lambda\mu^{-1}), \mathbf{L}_s^{(\mathrm{cl})}(\lambda) \otimes \mathbf{L}_{s'}^{(\mathrm{cl})}(\mu)\right]. \tag{1}$$

Here s, s' label the representations of $\widehat{osp}(1|2)$ and $\mathbf{r}$ is a classical $\mathbf{r}$-matrix. This relation gives the involutivity condition for the traces of the monodromy matrices and, therefore, the integrability. Our aim is to quantize expression (1) to obtain integrability in the quantum case. The quantum versions of the $\mathbf{L}^{(\mathrm{cl})}$ operators are expressed in terms of vertex operators (the Wick ordered exponentials of free bosonic fields) and fermionic free fields. The surprising fact is that one term is absent in the P-exponent present in the quantum version of $\mathbf{L}^{(\mathrm{cl})}$

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matrices, which we will denote $\mathbf{L}^{(\mathrm{q})}$. These $\mathbf{L}^{(\mathrm{q})}$ operators satisfy the so-called RTT relation:

$$\mathbf{R}_{ss'}(\lambda\mu^{-1})\Big(\mathbf{L}_s^{(\mathrm{q})}(\lambda)\otimes\mathbf{I}\Big)\Big(\mathbf{I}\otimes\mathbf{L}_{s'}^{(\mathrm{q})}(\mu)\Big)=\Big(\mathbf{I}\otimes\mathbf{L}_{s'}^{(\mathrm{q})}(\mu)\Big)\Big(\mathbf{L}_s^{(\mathrm{q})}(\lambda)\otimes\mathbf{I}\Big)\mathbf{R}_{ss'}\big(\lambda\mu^{-1}\big),$$

where $\mathbf{R}$ is a quantum R-matrix associated with quantum superalgebra $\widehat{osp}_{\mathrm{q}}(1|2)$. Considering the RTT relation in the first non-trivial representation of $\widehat{osp}_{\mathrm{q}}(1|2)$ we will obtain that the traces of monodromy matrices, that is the traces of shifted $\mathbf{L}^{(\mathrm{q})}$ operators, are commuting. Thus, we get the integrability condition in the quantum case. Considering the higher-dimensional irreducible representations, using the first orders of expansion in λ, we will obtain the functional relations for their eigenvalues. In such a way the introduced constructions give us the possibility of simultaneous diagonalization of the infinite family of IM. This allows us to use the obtained integrable structure as an integrable structure of superconformal field theory (SCFT). One can classify the states via IM eigenvalues. The functional relations for transfer matrices can be transformed to the thermodynamic Bethe ansatz equations, used in the theory of 'massless S-matrix' for the minimal models of SCFT. The SCFT minimal models (and the associated statistical systems at the critical point) have enough symmetries to classify the states in the model and to compute their correlation functions. However, when the system is perturbed by the operator commuting with IM, the IM are preserved but the (super)conformal symmetry is lost. In this case we can use SCFT symmetries to construct basic structures, such as the RTT relation and the transfer matrix relations, then we can study the integrable perturbed model by the proposed methods. Thus the quantization considered for the sKdV system can be used for different models, e.g. in the study of integrable perturbed SCFT and associated integrable statistical models off the critical point.

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